REPRESENTATION OF TIME DEPENDENT CHARACTERISTICS OF METALS

by

S. R. BODNER and Y. PARTOM
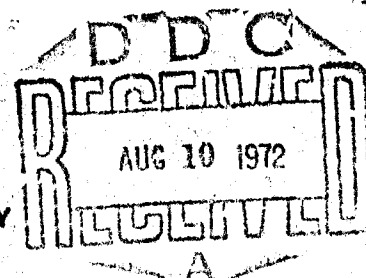
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Department of Materials Engineering

Technion - Israel Institute of Technology

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Representation of Time Dependent

Characteristics of Metals[1]

by

S.R. Bodner[2] and Y. Partom[3]

Abstract

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13. ABSTRACT

A new approach to the representation of time dependent inelastic material behavior is described. Realistic properties such as strain hardening, strain rate effects, and anelasticity can be incorporated in this description which is particularly well suited for the computational solution of structural problems involving cyclic loading and large inelastic strains. Application to technological metals such as titanium is indicated.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
VISCOPLASTICITY						
STRAIN RATE EFFECTS						
CONSTITUTIVE EQUATIONS						
STRAIN HISTORY EFFECTS						
TITANIUM						

Introduction

The analysis of the mechanical response of structures and machine parts in the range where the material response is inelastic and time dependent requires adequate representation of the material behavior under those conditions. The classical idealizations of elasticity, plasticity, and viscoelasticity have considerable limitations when time effects combined with strain hardening and inelasticity are significant factors. These limitations are especially severe when the structures are subjected to complicated loading histories that include changes of direction and rate of loading such as cyclic loading in the inelastic range. The various generalizations of the classical material idealizations that have been proposed to account for certain material properties, e.g. rate dependent plasticity, [1], [2], are difficult to use in structural problems and do not properly represent material response for general loading and unloading histories.

The present paper reports on a new method of characterization of material behavior that can serve for a wide range of properties including strain hardening, strain rate sensitivity, anelasticity, accumulation of large plastic strains, and creep. The method is well suited for the computer solution of structural problems involving large deformations and complicated loading histories. An interesting aspect of the representation is that the stress strain curve of the material is

a consequence of the constitutive equations and the conditions of loading. That is, the stress-strain curve is the solution of a particular boundary value problem and is not a "basic" material property.

A description of this approach has appeared in an earlier paper [3] for the case of perfect plasticity, i.e. neglect of strain hardening. The present paper reviews the procedure including the consideration of strain hardening. Examples are given to show the applicability of the equations to the case of titanium tensile specimens subjected to uni-axial loading at various uniform and changing strain rates.

General Formulation

The essential point in the present procedure is that the deformation
rate tensor d_{ij} is considered to consist of both elastic (fully reversible)
and inelastic (irreversible) components at each stage,

$$d_{ij} = d_{ij}^e + d_{ij}^p \qquad (1)$$

The relations between these components and the elastic stress, which is
the reference state variable, are the basic constitutive equations of
the material. There is therefore no distinct region of material res-
ponse that is fully elastic since inelastic strains would be present
at all stages of loading and unloading. A special unloading criterion
is therefore not required since the same constitutive equations hold
under all conditions. This makes the method particularly well adopted
for computer applications involving arbitrary loading histories.

Another consideration is that the total stress contains an anelas-
tic component in addition to the elastic stress. The anelastic stress
is introduced to account for viscous resistance to motion and is
responsible for energy losses for geometrically reversible motions,
e.g. internal damping. This stress can, in general, be expressed
as a function of the elastic stress and the total deformation rate.
The anelastic stress will, however, be taken as zero in the examples

discussed in this paper.

The equations relating the deformation rate to the velocity gradients and the strain rate using the Almansi strain measure have been described for general deformation states [3]. The elastic strain is a function of the elastic stress, so the elastic component of the deformation rate can, upon integration, be directly related to the elastic stress.

The constitutive law for the plastic (irreversible) component is also a relation between d^p_{ij} and σ_{ij}. In following the flow law of classical plasticity, this relation is taken to have the general form

$$d^p_{ij} = \underline{d}^p_{ij} = \lambda \underline{\sigma}_{ij} \tag{2}$$

where the bar symbol refers to the deviatoric tensor. The quantity λ is determined by squaring (2) to give

$$\lambda^2 = \underline{D}^p_2 / \underline{J}_2 \tag{3}$$

where $\underline{J}_2$ is the second invariant of the elastic stress deviator and $\underline{D}^p_2$ is the second invariant of the plastic deformation rate tensor. The von Mises yield criterion of classical plasticity states that plastic flow occurs when, in the present notation,

$$\underline{J}_2 = - Y^2/3 \tag{4}$$

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where Y is the yield stress in tension.

The present viscoplastic theory considers that a relation exists

between $\underline{D}^p_2$ and $\underline{J}_2$ to be used in conjunction with (2) and (3). This rela-

tion between the plastic deformation rate and the elastic stress invar-

iants,

$$\underline{D}^p_2 = f(\underline{J}_2) \tag{5}$$

therefore forms the constitutive equation for describing the viscoplas-

tic deformation properties of the material. Expressions for this rela-

tion are motivated by the equations relating dislocation velocity

with stress which are basic to the field of "dislocation dynamics",

e.g. [4]. A useful particular form for (5) is

$$D^p_2 = D^2_o \exp\left(-[C^2/(-\underline{J}_2)]^n\right) \tag{6}$$

where

$$C^2 = \frac{1}{3} Z^2 \left(\frac{n+1}{n}\right)^{1/n} \tag{6a}$$

In these equations, D_o, Z and n are material parameters. The coeffic-

ient D_o is the asymptotic value of the deformation rate at large

stresses, i.e. the plastic deformation rate is bounded. The quantity

n is a measure of the steepness of the curve and is therefore a measure

became very large. That would correspond to an upward turning on the stress
strain curve at large strains which is not realistic. Microscopic analy-
ses also indicate that plastic flow never fully ceases since there are lim-
its to the distances between the obstacles that oppose dislocation motion.
Strain hardening can therefore be introduced into (6) by making D_o a
decreasing function or C an increasing function of W_p. The latter was
chosen in the present example and the parameter Z was taken to have the
form

$$Z = Z_1 + (Z_o - Z_1) \ e^{-mW_p/Z_o} \qquad (8)$$

where Z_1, Z_o, and m are new material parameters.

This formulation of strain hardening corresponds to isotropic
hardening which means that it would not account for any Bauschinger
effect. Th would require introducing particular non-symmetric features
into the analysis which is possible in principle but very complicated.
The above formulation could also account for other special material
eff   such as age hardening and strain ageing. Age hardening would mean
that d_{ij}^p would be a decreasing function of absolute time. Strain ageing
is a more complicated phenomenon and could be considered by making d_{ij}^p
decrease with the time of deformation. This would mean that shorter
deformation times would correspond to larger values of d_{ij}^p and therefore
to lower stresses which are the macroscopic characteristics of strain
ageing. The present example, however, considers only strain hardening as
indicated by (8).

Specialization to Uniaxial Straining

The examples described in this paper are of uniaxial straining of a material at various uniform and changing strain rates. The constitutive equations developed for this case can also be used for multiaxial boundary value problems.

The deformation rate tensor is defined in terms of the velocity gradient,

$$d_{ij} = 1/2(v_{i,j} + v_{j,i}) \tag{9}$$

where v_i is the velocity vector. For the uniaxial stress state, the only non-zero deformation rate components are d_x (axial direction) and d_y in both transverse directions. All shearing components vanish for this case. The axial deformation rate is simply

$$d_x = \frac{dv_x}{dx} \tag{10}$$

where v_x is the particle velocity in the axial direction. In terms of the crosshead velocity of the straining device V_c and the specimen gauge length L,

$$d_x = V_c/L \tag{11}$$

The other component d_y is determined by the state of stress. The deviatoric components of the deformation rate are then given by

$$\underline{d}_x = 2/3 \ (d_x - d_y) \tag{12a}$$

$$\underline{d}_y = \underline{d}_z = - 1/3 \ (d_x - d_y) \tag{12b}$$

and each component is considered to be the sum of elastic and plastic parts, e.g.

$$\underline{d}_x = \underline{d}_x^e + \underline{d}_x^p \tag{13}$$

The elastic stress-strain relations, assuming the elastic strains are sufficiently small so that Hooke's Law is applicable, are

$$\underline{\sigma}_{ij} = 2G \ \underline{\varepsilon}_{ij} \tag{14}$$

$$\sigma_{kk} = 3K \ \varepsilon_{kk} \tag{15}$$

and in this example σ_x is the only non-zero stress component.

For large strains, the deformation rate is not, in general, equal to the strain rate [3]. However, this identity does hold for the simple geometry of the present problem since the other terms in the general relationship become zero.

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The elastic part of the deformation rate tensor is related to the stresses through (14) and (15) and the plastic part through (2), (3), (6) and (8). The material is compressible for elastic deformations (15) and is incompressible for plastic deformations in accordance with the flow law (2). The rate of plastic work, W_p, is given by

$$\dot{W}_p = \sigma_x d_{x-x}^p \qquad (16)$$

These equations can then serve to determine the stress required to pull a rod of the material at a uniform velocity V_c. This is actually a particular boundary value problem whose solution leads to the uniaxial force-elongation (stress-strain) relation of the material.

A numerical scheme was devised to compute the stress from the preceding equations when the material constants and the applied velocity V_c are given. The method is a step by step procedure which follows the deformation history. All quantities such as d_{ij}^e, d_{ij}^p, W_p, the total elongation, and the stress are determined at each step. The numerical scheme can be readily adjusted to account for changes in the applied velocity and for loading and reloading. That is, the method can consider completely arbitrary loading or straining histories. In this paper, however, only examples involving uniform velocities and a single change of velocity are described

Application to Titanium

A series of tensile tests were performed in a 10 ton capacity Instron testing machine on specimens of commercially pure titanium. The specimens were cut from a 1 mm thick plate in the rolling direction and were 8 mm wide. An extensometer was used for the strain measurement and the load was recorded as a function of strain. Titanium is a fairly rate sensitive material which makes it useful for studying the effect of different straining rates and the response to changes of rate during a test. In general, material response is influenced by the complete strain rate history and titanium appears to be a good specimen material for such studies. This has been emphasized recently by a number of investigators, e.g. [6]. The proposed method of material representation and the associated constitutive equations intrinsically include strain rate history effects.

In order to examine the applicability of the present theory to titanium, the material constants of the constitutive equations, (6), (6a), (8) were determined from the results of two tests at different constant extension rates. The response to other straining rates and to varying straining histories were then calculated and compared to corresponding experimental results

Tests were conducted for four constant crosshead velocities: 0.005, 0 05, 0.5 and 1.0 cm/min. The effective overall specimen gauge length was 52 mm so the imposed velocities correspond respectively to the strain

rates: 1.6×10^{-5}, 1.6×10^{-4}, 1.6×10^{-3}, and $3.2 \times 10^{-3} sec^{-1}$.

The material constants were obtained by fitting the calculated response of the material at the highest and lowest rates to the corresponding experimental curves. The values of the material constants determined in this manner for commercially pure titanium are:

$$Z_o = 11.5 \text{ Kbars } (112.8 \text{ Kg/mm}^2)$$

$$Z_1 = 14.0 \text{ Kbars } (137.0 \text{ Kg/mm}^2)$$

$$D_o^2 = 10^8 \text{ sec}^{-2}$$

$$n = 1$$

$$m = 100$$

The elastic constants for titanium are

$$K = 1.23 \times 10^3 \text{ Kbars } (12.0 \times 10^3 \text{ Kg/mm}^2)$$

$$G = 0.44 \times 10^3 \text{ Kbars } (4.3 \times 10^3 \text{ Kg/mm}^2)$$

The calculated stress-strain curves for these constants are shown in Fig. 1 for the highest and lowest straining rates. Also shown are the experimental curves to which they were fitted. Calculated stress-strain curves for the other straining rates are shown in Fig. 2 along with the corresponding experimental results.

Of greater interest is the effect of varying strain and strain rate histories on the deformation characteristics. One significant experiment of this kind is to change the crosshead velocity during the course of a test. This can be easily accomplished on an Instron machine by pressing

the button that activates a magnetic clutch on the speed regulator. A number of tests were run in which the slowest and fastest rates were interchanged at 4% strain without unloading. The experimental results were consistently reproducible, Figs. 3, 4, and could be summarized as follows:

(a) Immediately upon changing from the high to the low rate, the stress drops in an essentially elastic manner to about or slightly above the level corresponding to the lower rate for a constant rate test. The stress then shows a small rise and continues approximately parallel and above the constant rate curve and tends toward it with increasing strain (Fig. 3).

(b) Upon changing from a lower to a higher rate, the immediate response is close to the elastic value and the stress then approaches the curve corresponding to the higher uniform rate test but at a lower level. There is a small rise and fall of the stress curve after the initial elastic response which is similar to the "upper yield point" phenomena. The stress tends toward the uniform rate curve with increasing strain. The flow stress at a high rate is therefore less when it is subjected to prior deformation at a low rate than if uniformly strained at the high rate (Fig. 4).

Another closely related experiment would be to unload the specimen at a given strain and then to reload at a different rate. A few experiments of this kind were performed and the results indicated little overall differences between this case and that of rate changing without unloading.

The "cusp" observed in going from the high to the low rate in the former tests is not observed when the specimen is fully unloaded before the rate is changed. An "upper yield point" effect is also observed in this case upon reloading at a higher rate, but it is less pronounced than when the rate is changed without unloading.

Similar experiments to type (b) above, namely changing from a low to a high rate without unloading have been performed on titanium in shear [6] · generally similar results. The "upper yield point" effect was, however, not observed in those tests [6]. An experiment of this kind on aluminum for a very large change of rate of loading has been reported [7] and the "upper yield point" behavior of the incremental response was observed. Various experiments on changing the rate of straining after complete unloading were performed on aluminum [8,9] and led, in general, to results similar to those obtained here.

The calculated response of the material based on the present theoretical formulation for the same variable strain rate history gave results that closely approximated the experimental ones, Figs. 3, 4. The "cusps" observed on reducing the strain rate and the "upper yield point" observed on increasing the rate were, however, not reproduced in the calculated response curves. These seem to be transient effects which depend on more detailed mechanisms of plastic flow than are represented in the present theoretical formulation. It may be possible to include such effects by

generalizing the material constants to more closely simulate microscopic para-
meters (internal variables) such as dislocation density and velocity. The
reason for the respective stress levels upon changing rates can be explained
in terms of the plastic work W_p prior to the rate change which influences
the subsequent flow stress. W_p is larger at the higher rate which leads
to a relatively higher stress curve upon reducing the rate (compared to a
constant lower rate test), while the reverse holds for the other case.
These stress level differences could also be explained in terms of the
developed microstructure but this will be left to a subsequent paper.

It is particularly interesting to examine the details of the deforma-
tion upon changing from the lower to the higher rate. For this particular
case, the plastic deformation rate component d_{ij}^p is initially 99.7% of
the total d_{ij} at 4% strain. Immediately after the change, the value of
d_{ij}^p increases slightly but its percentage of d_{ij} drops to 56.6%. The in-
cremental response has therefore a large elastic component and experimentally
the response may appear to be fully elastic for approximate measurements.
If the change of imposed velocity at the specimen end had been sufficiently
rapid to generate waves, then an elastic wave would propagate along the
specimen. The plastic component would not be dominant and would attenuate
rapidly with distance. Observations some distance from the end would indi-
cate that the incremental response to the velocity change is elastic.

The proposed constitutive equations are also suitable for cyclic
loading histories, which would be important for low cycle fatigue studies.

An independent criterion, however, would have to be introducted to indi-
cate the onset of fatigue microcracks or other failure phenomena. If
such a criterion were expressible in terms of the state variables σ_x and
W_p and other quantities such as accumulated plastic strain, then the
present analysis could serve to determine the condition for which the
criterion would be reached for very general cyclic loading histories.

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List of Captions

Fig. 1 - Experimental and Calculated (Fitted) Stress-Strain Curves for Titanium at Constant Strain Rates.

Fig. 2 - Experimental and Calculated (Derived) Stress-Strain Curves for Titanium at Constant Strain Rates.

Fig. 3 - Experimental and Calculated (Derived) Stress-Strain Curves for Titanium Subjected to a Rapid Change (Decrease) in Strain Rate.

Fig. 4 - Experimental and Calculated (Derived) Stress-Strain Curves for Titanium Subjected to a Rapid Change (Increase) in Strain Rate.
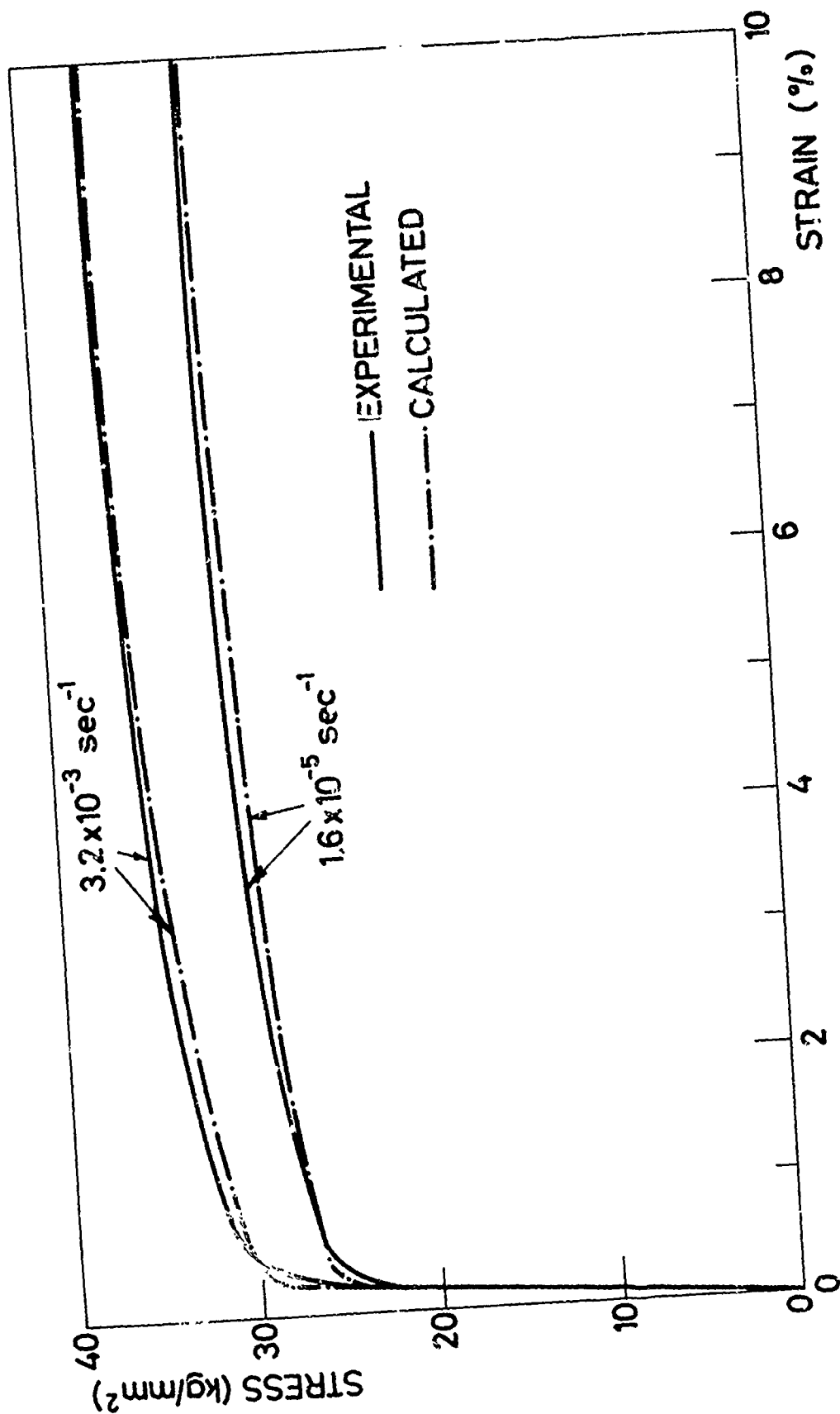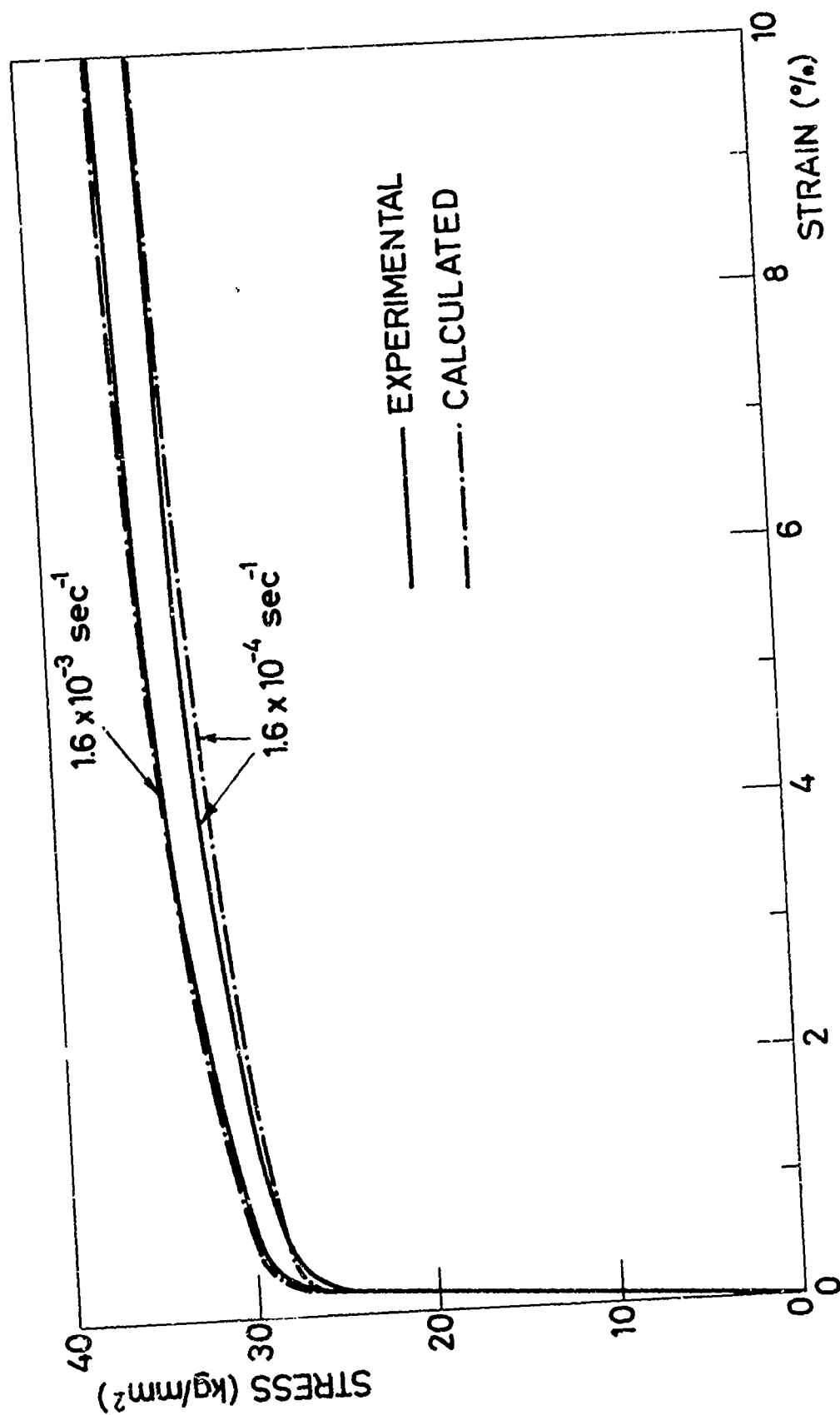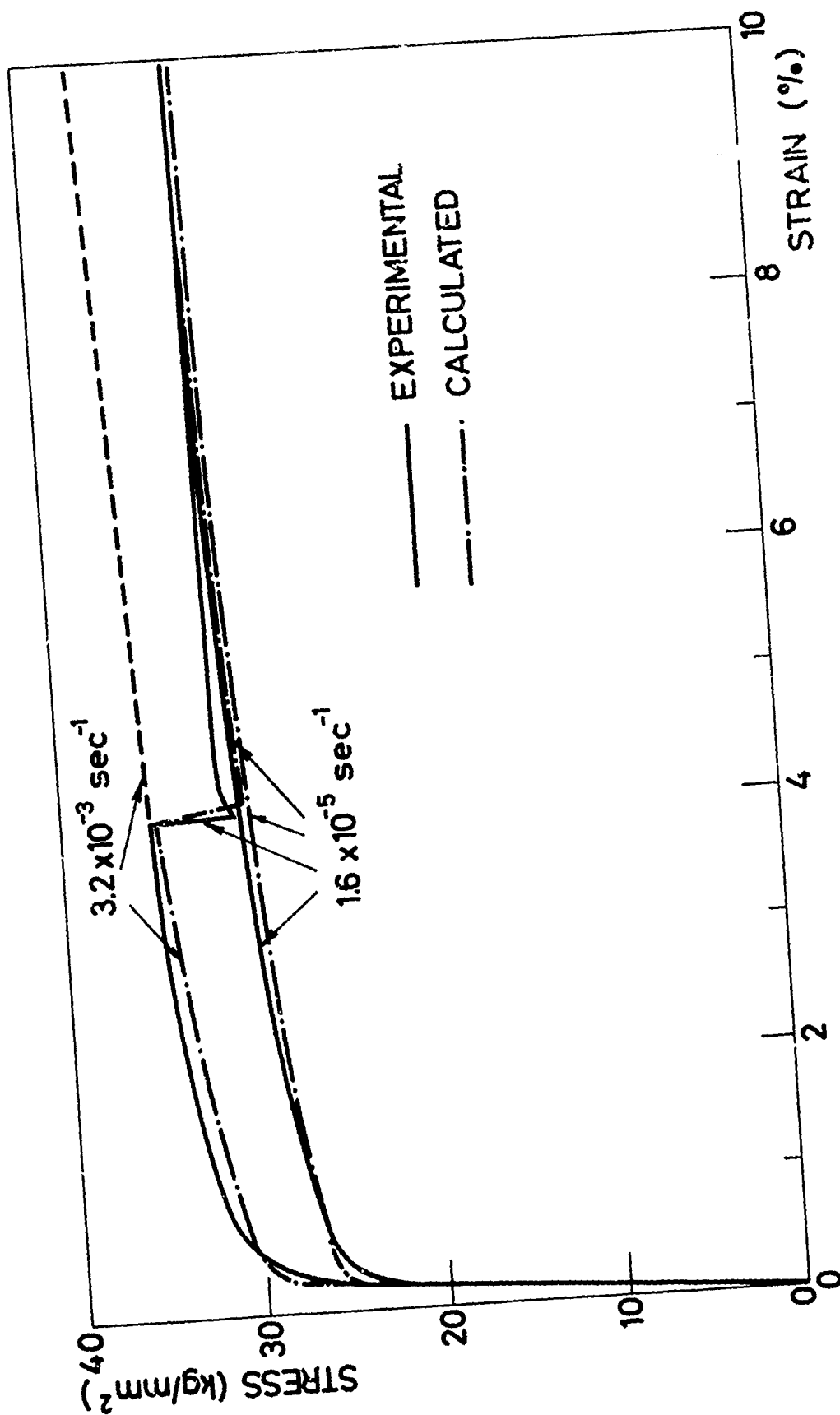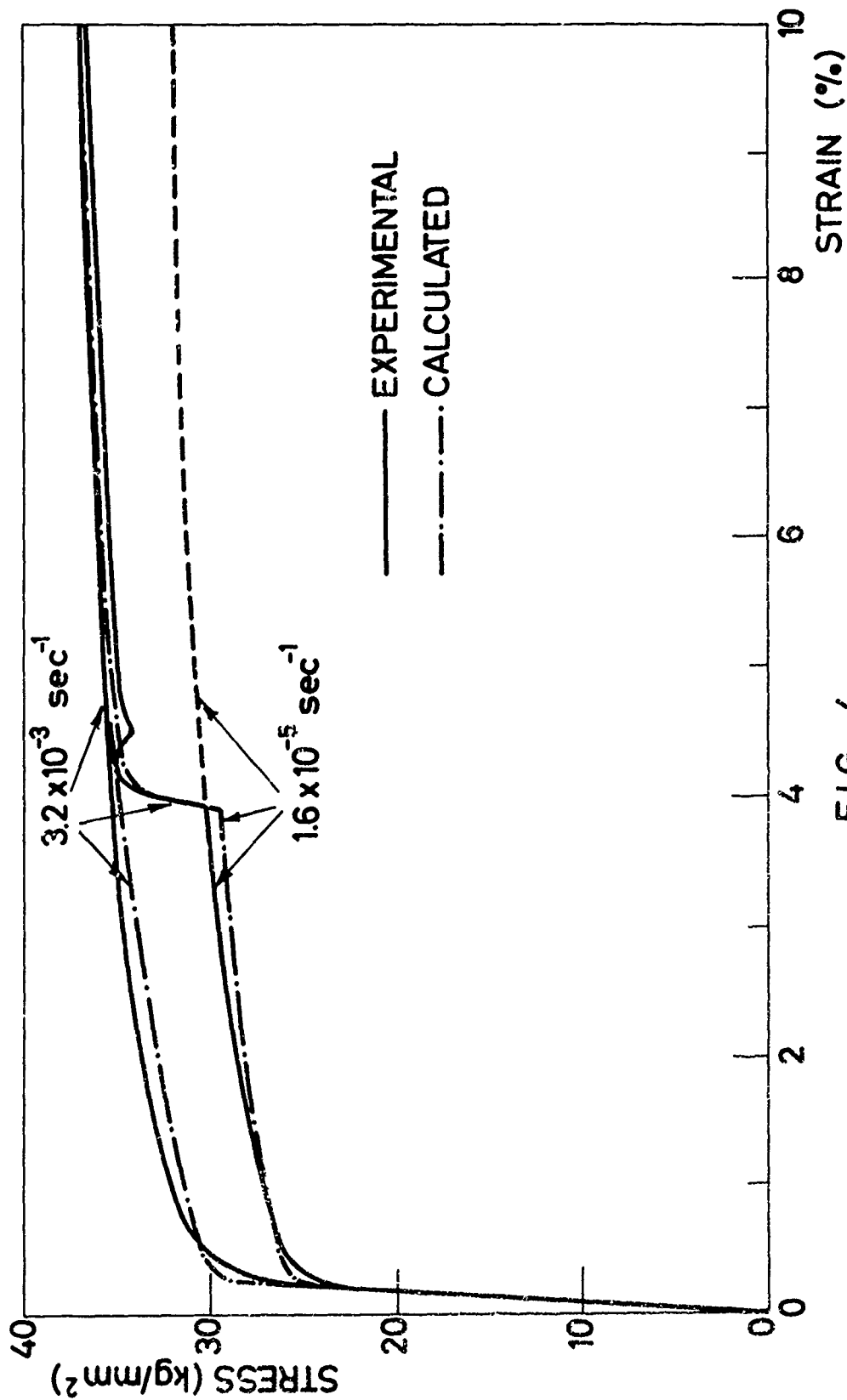
FIG. 1

STRAIN (%)

STRESS (kg/mm²)

3.2x10⁻³ sec⁻¹

1.6x10⁻⁵ sec⁻¹

EXPERIMENTAL

CALCULATED

F I. G. 2

FIG 3

STRAIN (%)

STRESS (kg/mm²)

EXPERIMENTAL
CALCULATED

3.2×10^{-3} sec^{-1}

1.6×10^{-5} sec^{-1}

FIG. 4